# Bubbles and crises

## ECON4335 Lecture 9

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# Main source

A. Martin and J. Ventura: "Theoretical notes on bubbles and the current crisis". ECB 2011
Sections 3 and 4 can be skipped
Combines bubbles and financial frictions

# Outline

- Martin-Ventura model
- Equlibrium without bubbles
- Effects of financial frictions
- Equilibrium with rational bubbles
- When bubbles break
- Rational bubbles vs irrational exuberance

# Consumers

- Live for two periods
- Each supplies one unit of labor in the first period
- Save all wage income for consumption in the second period
- Are risk neutral
- Share $\varepsilon$ are entrepreneurs

# New and old firms

- Entrepreneurs: Invest in new firms, borrow
- Non-Entrepreneurs: Invest in old firms, lend to entrepreneurs
- Entrepreneurs are more efficient in investment

A real investment $z_{j,t}$, yields

- for a non-entreprenueur: $z_{j,t}$ units of capital
- for an entrpreneur: $\pi_t z_{j,t}$ units of capital where $\pi_t > 1$

Capital accumulates according to

$$k_{j,t+1} = z_{j,t} + (1-\delta)k_{j,t}, \quad \text{for old firm} \tag{1}$$

$$k_{j,t+1} = \pi_t z_{j,t}, \quad \text{for new firm} \tag{2}$$

Only old firms produce final goods. Production functions:

$$y_{j,t} = \ell_{j,t}^{1-\alpha} k_{j,t}^{\alpha} \tag{3}$$

# The fundamental value of a firm

- The fundamental value of a firm is the value of the capital stock that belongs to the firm

$$V_{j,t} = (1 - \delta)k_{j,t} \tag{4}$$

- $V_{j,t}$ is the price of an old firm in period $t$ after production has been carried out and depreciation has taken place, but before the new owner has added his new real investments to the capital stock

# Some macro relations

Per worker production function in macro

$$y_t = \ell_t^{1-\alpha} k_t^{\alpha} = k_t^{\alpha} \tag{5}$$

$y_t$, $k_t$ and $\ell_t$ are per worker, $\ell_t = 1$
Since non-entrepreneurs demand the same gross return, $R_{t+1}$ ,from buying shares in old firms and lending to entrepreneurs;

$$R_{t+1} = 1 + \alpha k_{t+1}^{\alpha-1} - \delta = \alpha k_{t+1}^{\alpha-1} + 1 - \delta \tag{6}$$

Labor is paid its marginal product

$$w_t = (1-\alpha) k_t^{\alpha} \tag{7}$$

# Rate of return on share in old firm

For the record, can be skipped
Revenues/Outlays for investor who buys firm:

$$\begin{aligned} \frac{y_{t+1} - w_{t+1} + V_{t+1}}{V_t + k_{t+1} - k_t(1-\delta)} &= \frac{k_{t+1}^{\alpha} - (1-\alpha)k_{t+1}^{\alpha} + k_{t+1}(1-\delta)}{k_t(1-\delta) + k_{t+1} - k_t(1-\delta)} \\ = \frac{\alpha k_{t+1}^{\alpha} + (1-\delta)k_{t+1}}{k_{t+1}} &= \alpha k_{t+1}^{\alpha-1} + 1 - \delta = R_{t+1} \end{aligned} \tag{8}$$

When firms are bought and sold at their fundamental values, the investor gets the real interest rate in return

# Entrepreneurs: The Credit Constraints

Borrowing constraint:

$$\underbrace{R_{t+1} f_{j,t}}_{a} \leqq \underbrace{\Phi_{t+1}}_{b}[\underbrace{\alpha k_{t+1}^{\alpha-1}}_{c} \underbrace{\pi_t}_{d} \underbrace{(f_{j,t} + w_t)}_{e} + \underbrace{V_{j,t+1}}_{f}] \tag{9}$$

a) Repayment on loan $f_{j,t}$

b) Financial friction $\Phi_{t+1} < 1$

c)-f) Gross return next period

c) Marginal product of capital $(= R_{t+1} - (1 - \delta))$

d) Efficiency of investment

e) Amount invested = loan + wage income

f) $V_{t+1}$ revenue from sale of firm

Since $\pi_t > 1$ entrepreneur can always get higher return than non-entrepreneur $\Longrightarrow$Entrepreneur will always borrow as much as possible.

If $\Phi_{t+1}\pi_t < 1$, borrowing constraint will be effective.

# Entrepreneurs: The Credit Constraints

$$R_{t+1} f_{j,t} = \Phi_{t+1}[(R_{t+1} - 1 + \delta) \underbrace{\pi_t (f_{j,t} + w_t)}_{k_{j,t+1}} + V_{j,t+1}] \qquad (10)$$

$$R_{t+1} f_{j,t} = \Phi_{t+1} \left[R_{t+1} \pi_t (f_{j,t} + w_t) - (1-\delta) k_{j,t+1} + (1-\delta) k_{j,t+1}\right]$$

$$R_{t+1} f_{j,t} = \Phi_{t+1} R_{t+1} \pi_t f_{j,t} + \Phi_{t+1} R_{t+1} \pi_t w_t$$

Solve for $f_{j,t}$:

$$f_{j,t} = \frac{\Phi_{t+1} \pi_t}{1 - \Phi_{t+1} \pi_t} w_t > 0 \quad \textit{for} \quad \Phi_{t+1} \pi_t < 1 \qquad (11)$$

- Borrowing is proportional to wage income
- Constraint is effective only when financial frictions are strong relative to the efficiency advantage of entrepreneurs

# Dynamics of the economy's capital stock.

$$k_{t+1} = w_t + \varepsilon(\pi_t - 1)z_{j,t} \tag{12}$$

$k_{t+1}$ =wages (savings)
+ gain from entrepreneurs doing part of the investment
Typical entrepreneur invests

$$z_{j,t} = f_{j,t} + w_t = \frac{\Phi_{t+1}\pi_t}{1 - \Phi_{t+1}\pi_t} w_t + w_t = \frac{1}{1 - \Phi_{t+1}\pi_t} w_t$$

Inserted in 12

$$k_{t+1} = \left[1 + \frac{\varepsilon(\pi_t - 1)}{1 - \Phi_{t+1}\pi_t}\right](1 - \alpha)k_t^{\alpha} \tag{13}$$

# Effects of shocks

$$k_{t+1} = \left[1 + \frac{\varepsilon(\pi_t - 1)}{1 - \Phi_{t+1}\pi_t}\right](1-\alpha)k_t^{\alpha}$$

- More efficient entrepreneurs (higher $\pi_t$) $\rightarrow$more growth
- Less financial frictions (higher $\Phi_t$) $\rightarrow$more growth

With $\pi_t$and $\Phi_{t+1}$constant, economy goes to steady state characterized by $k_{t+1} = k_t = k_*$ or

$$\alpha k_*^{\alpha-1} = \frac{\alpha}{1-\alpha} \cdot \frac{\epsilon(\pi_* - 1)}{\epsilon(\pi_* - 1) + 1 - \Phi_*\pi_*} \tag{14}$$

- Higher $\pi$ $\rightarrow$More capital
- Higher $\Phi$ $\rightarrow$More capital

# Bubbles

Value of firm with bubble:

$$V_{j,t} = (1-\delta)k_{j,t} + b_{j,t} \tag{15}$$

$(1-\delta)k_{j,t}$ = fundamental, $b_{j,t}$ =bubble

- Non-entrepreneurs have to pay more for firms
- Entrepreneurs get more for new firms and get more credit
- Some savings are diverted from real investment to consumption of the old
- More real investment is undertaken by the most efficient investors
- Total effect on real investment ambiguous

# Requirements for a rational bubble

- Bubble must have expected return equal to interest rate

$$\frac{E_t b_{j,t+1}}{b_{j,t}} = E_t R_{t+1} \tag{16}$$

- Bubble should never become too large for the young to purchase

(17)

# Bubbles and the credit constraint

New firm at $t$, value at $t+1$:

$$V_{j,t+1} = (1-\delta)k_{j,t+1} + E_t b^N_{j,t+1} \tag{18}$$

The basis for getting loans is augmented by the present value of the bubble:

$$f_{j,t} = \frac{\Phi_{t+1}\pi_t}{1-\Phi_{t+1}\pi_t}\left[w_t + \frac{E_t b^N_{j,t+1}}{R_{t+1}}\right] \tag{19}$$

Total investment in new firm

$$z_{j,t} = f_{j,t} + w_t = \frac{1}{1-\Phi_{t+1}\pi_t}w_t + \underbrace{\frac{\Phi_{t+1}\pi_t}{1-\Phi_{t+1}\pi_t}\cdot\frac{E_t b^N_{j,t+1}}{R_{t+1}}}_{a} \tag{20}$$

a: additional loan and investment because of bubble.

# Capital accumulation

$$k_{t+1} = w_t \underbrace{-b_t - b_t^N}_{a} + \frac{\varepsilon(\pi_t - 1)}{1 - \Phi_{t+1}\pi_t} w_t + \underbrace{\frac{\varepsilon(\pi_t - 1)}{1 - \Phi_{t+1}\pi_t} \cdot \frac{E_t b_{j,t+1}^N}{R_{t+1}}}_{b} \tag{21}$$

a: savings go to buy bubbles instead of real investment
b: entrepreneurs get to do more of the investment

$$k_{t+1} = \left[1 + \frac{\varepsilon(\pi_t - 1)}{1 - \Phi_{t+1}\pi_t}\right](1-\alpha)k_t^{\alpha} + \frac{\varepsilon(\pi_t - 1)}{1 + \Phi_{t+1}\pi_t} \cdot \frac{E_t b_{j,t+1}^N}{\alpha k_{t+1}^{\alpha-1} + 1 - \delta} - b_t - b_t^N \tag{22}$$

Evolution of aggregate bubble

$$E_t b_{t+1} = R_{t+1}(b_t + b_t^N) = (\alpha k_{t+1}^{\alpha-1} + 1 - \delta)(b_t + b_t^N) \tag{23}$$

# Bubbly episodes - an example

- Probability of bubble ending in period t constant equal to $p$
- Bubble starts with $b_t^N = b^N > 0$
- While bubble goes on $b_t^N = nb_t, \quad n > 0$
- Probability of a second bubble starting is negligible
- Auxiliary assumptions:

$$\delta = 1, \quad \Phi_t = \Phi, \quad \pi_t = \pi \tag{24}$$

# The dynamics of the aggregate bubble

Expected growth in aggregate bubble comes both from old and new firms:

$$E_t b_{t+1} = R_{t+1}(1+n)b_t \tag{25}$$

Since bubble breaks with probability $p$

$$E_t b_{t+1} = p \cdot 0 + (1-p)b^c_{t+1} \tag{26}$$

where $b^c_{t+1}$ is value of bubble if it continues.
Combining the two equations gives

$$b^c_{t+1} = \left[R_{t+1}(1+n)/(1-p)\right] b_t \tag{27}$$

- A bubble that continues grows faster than the interest rate

# Dynamics of bubble continued

Define share of bubble in savings $x_t = b_t/w_t$
If bubble continues:

$$x_{t+1} = \frac{b^c_{t+1}}{w_{t+1}} = \frac{R_{t+1}w_t}{w_{t+1}} \cdot \frac{1+n}{1+p}x_t \tag{28}$$

By 1. order conditions $R_{t+1}/w_{t+1} = \alpha/(1-\alpha)k_{t+1}$. Hence

$$x_{t+1} = \frac{\alpha}{1-\alpha} \cdot \frac{1+n}{1-p} \cdot \frac{w_t}{k_{t+1}}x_t \tag{29}$$

$k_{t+1}/w_t$ can be found from (21). Result

$$x_{t+1} = \frac{\frac{\alpha}{1-\alpha} \cdot \frac{1+n}{1-p}x_t}{1 + \frac{\varepsilon(\pi-1)}{1-\Phi\pi} + \left(\frac{(\pi-1)\Phi n}{1-\Phi\pi} - 1\right)(1+n)x_t} \tag{30}$$

Single equation determining evolution of bubble until it collapses.
Share independent of $k_t$.

# Path for capital stock

From (21) and definition of $x_t$:

$$k_{t+1} = \left[1 + \frac{\varepsilon(\pi - 1)}{1 - \Phi\pi} - (1 - \frac{\Phi(\pi - 1)n}{1 - \Phi\pi})(1 + n)x_t\right](1 - \alpha)k_t^{\alpha} \tag{31}$$

Two types of bubbles:

- Contractionary

$$\frac{\Phi(\pi - 1)n}{1 - \Phi\pi} < 1$$

  Bubbles reduce capital stock and raise interest rates

- Expansionary

$$\frac{\Phi(\pi - 1)n}{1 - \Phi\pi} > 1$$

  Strong increase in loans to entrepreneurs, many new bubbles. Bubbles raise capital stock, lowers interest rates.

# Contractionary bubble

$$\frac{\Phi(\pi-1)n}{1-\Phi\pi} < 1$$

- Denominator in the equation for the evolution of the bubble share (30)may be negative if $x_t$ is high.
- Negative denominator signals that the bubble is already too high to be absorbed by the investors.
- The initial value of a rational bubble must below $\bar{x}$ where $\bar{x}$ is the value of $x_t$ that makes the denominator in (30 ) zero.
- A stationary equilibrium for $x_t$ with a bubble share, $0<x_* < \bar{x}$ may exist, but in that case it is unstable.
- The only sustainable bubbles are those that are expected to decrease relative to GDP over time.
- This means the (net) interest rate has to be below the growth rate of the economy, or below zero in our example.
- Even this kind of bubble can burst, but there is no expansion of output before that.

Les relevant for current crisis

# Expansionary bubble

$$\frac{\Phi(\pi - 1)n}{1 - \Phi\pi} > 1$$

Bubbles lead to strong credit expansion

- Made possible by financial frictions
- A stable stationary equilibrium for $x_t$ with $0 < x_* < 1$ may exist.
- Bubbles may start low and grow for a long time
- AS the bubbles grow, interst rate goes down

More relevant for current crisis